INTERRUPTED WAVE SYNCHRONISATION

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ABSTRACT. In this paper the phenomenon of interrupted wave synchronisation of an automatic phase control circuit has been studied. The response of such a circuit preceded by a tunable high Q circuit to an interrupted sinusoid has been analysed. Simple but not very accurate expressions for the locking range of the APC systems when the incoming signal is an interrupted one have been derived. When the APC circuit is not preceded by a tunable high Q circuit it has been found that the locking range increases with increasing values of the rate of interruption and duty cycle of the incoming wave. Necessary and sufficient conditions have been presented for sideband locking. It has been found that the locking of the local oscillator with any of the Fourier's Component becomes easy when the conventional APC circuit is preceded by a tunable high Q circuit. Experimental results regarding carrier and side-band locking range have been presented and found to be in good agreement with the conclusions of the analysis.

INTRODUCTION

Locking phenomena in an automatic phase control circuit with a CW signal have been studied by many authors (Richman, D., *et al* 1954). In this paper locking phenomena in an automatic phase control circuit with an interrupted continuous wave signal with a high value of carrier to noise ratio (CNR) will be examined. Direct synchronisation of a continuous wave oscillator with an interrupted sinusoid has, however, been studied in detail by Fraser, D. W. Injection synchronised oscillators, however, suffer from the defect that the independent control of the synchronisation range and the noise bandwidth of the system could not be achieved without affecting the stability of the local oscillator. As such method of indirect synchronisation—automatic frequency control (AFC) and automatic phase control (APC) of an oscillator were suggested. In an AFC system the frequency of the local oscillator is compared to a reference frequency and thus this system requires a finite error although small, in the frequency of the local oscillator for its required operation. The phase locked system (APC), on the other hand, requires no steady state error in the output frequency of the oscillator but instead utilises an error in the output phase which is the 'integral' of the controlled variable i.e. the output frequency. Thus, here these parameters can be controlled independent of one another.

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It has been shown that in the case of CW synchronisation an idea about the average network gain of the loop filter and maximum possible synchronising range gives a reasonable estimate of the locking range and locking time (Chakrabarti, N. B. *et al*, 1964). A typical block diagram of an APC circuit is shown in Fig. 1. The input to the system, in contrast to the case of CW synchronisation, consists of a single tone CW signal which is being interrupted at a suitable rate. The spectral

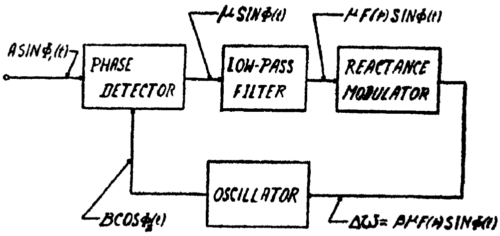

Fig. 1. A typical block diagram of an automatic phase control circuit incorporating a phase detector, a low-pass filter, a reactance modulator and an oscillator.

components of the input wave consist of a carrier and side-band components which are separated from the carrier frequency component by the frequency of interruption and its multiple thereof. From this one can easily conjecture that the local oscillator can be made to lock with any one of the Fourier Components (of couse with proper choice of system parameters). If the input is a stable one that is if a CW wave being interrupted is derived from a crystal oscillator and the waveform causing interruption is also stable, then oscillation with stable phase and frequency can be obtained by locking an oscillator or oscillators with the different sideband components. Such a technique permits saving of crystals in a frequency synthesis. Another very important use to which this phenomenon of interrupted wave synchronisation may be put is to convert a PRF mdulation into frequency modulation.

In section 2 the governing equations of an APC circuit with an interrupted CW signal as the input have been derived. In section 3 the simple APC circuit has been studied in detail and an equation for the locking range in terms of the duty cycle of the input wave and the system parameters has been derived.

Section 4 deals with the analysis of an APC circuit with a low-pass filter in the loop. In this section an expression for the locking ratio for the carrier frequency component has been presented.

Section 5 presents an analysis of this simple APC circuit when it is preceded by a tunable high Q circuit. Expressions for the locking range for the side-band components are also given.

Section 6 deals with the analysis of the APC circuit incorporating a low-pass filter in the loop and preceded by a high Q tuned circuit. This is followed by a description of the experimental set up and a discussion of the experimental results regarding carrier locking range and sideband locking in section 7. Experimental results are in good agreement with the conclusion of the analysis presented in the text.

DERIVATION OF THE LOOP EQATIONS

Let us consider the typical APC circuit, with a low-pass filter in the loop, for synchronisation with an interrupted sinusoid having a duty cycle of T/Tr (Fig. 1). During the on-period of the input wave the phase-detector acting as an error sensing device of the loop produces an output ($\mu \sin \phi$) that controls the instantaneous phase or frequency of the voltage controlled oscillator through the low-pass filter in the loop. Here μ is the sentitivity of the phase detector and ϕ is the instantaneous phase difference between the reference input and the local oscillator during the on-period. During the off-period the instantaneous phase or the frequency of the VCO is controlled by the stored energy in the low-pass filter in the loop during the on-period of the input wave. The governing equation of the APC loop for the on-period is given by

$$\frac{d\phi}{dt} = \Omega - KF(p) \sin \phi, \qquad \ldots \quad (2.1)$$

and that for off-period is given by

$$\frac{d\phi}{dt} = \Omega - \beta e_d(t), \qquad \ldots \quad (2.2)$$

where

$$e_d(t) = \int_0^t e(t_1) f(t-t_1) dt, \qquad \ldots \quad (2.3)$$

and β is the sensitivity of the reactance tube; $F(p)$ is the filter transfer-function and K is the maximum possible synchronising range in radian per second and and $e(t_1)$ is given by

$$e(t_1) = \mu \sin \phi(t_1) \qquad \ldots \quad (2.4)$$

and $f(t-t_1)$ is the impulse response of the filter network at time $t = t_1$.

THE SIMPLE APC CIRCUIT

The simple APC circuit consists of a phase detector and a voltage controlled oscillator. The governing equation of such a simple circuit is given by

$$\frac{d\phi}{dt} = \Omega - K \sin \phi, \; 0 \leq t \leq T \qquad \ldots \quad (3.1)$$

and

$$\frac{d\phi}{dt} = \Omega, \quad T \leqslant t \leqslant T_r \qquad \ldots \quad (3.2)$$

where the symbols have their usual significance. It is to be noted that locking can only occur when open loop frequency error is less than the maximum possible synchronising range K and the net phase shift between the local oscillator and the reference input is zero. Expressed analytically

$$\int^{T_r} \left(\frac{d\phi}{dt}\right) dt = 0 \qquad \ldots \quad (3.3)$$

$$\text{i.e.} \quad \int^{T} (\Omega - K \sin \phi)\, dt + \int^{Tr} \Omega \, dt = 0 \qquad \ldots \quad (3.3a)$$

which on comparison with Eq. (3.1) and on integration yield the following simple equation

$$2 \tan^{-1} \left[\frac{1}{x} - \frac{\sqrt{1-x^2}}{x} \tanh \left(Kt_0 \frac{\sqrt{1-x^2}}{2} \right) \right]$$

$$-2 \tan^{-1} \left[\frac{1}{x} - \frac{\sqrt{1-x^2}}{x} \tanh K(t_0+nTr) \frac{\sqrt{1-x^2}}{2} \right.$$

$$= x(1-n)KT, \qquad (3.4)$$

where x stands for the locking ratio Ω/K and n represents the duty cycle of the input interrupted wave and t_0 is the constant that determines the initial difference of phase between the local oscillation and the reference oscillation. The value of t_0 can be found from the fact that at the edge of the band of synchronisation it can be assumed with a fair degree of accuracy that the phase difference is very approximately equal to $\pi/2$ radians. Therefore, with this assumption one can write from Eq. (3.9)

$$\tanh \left[Kt_0 \frac{\sqrt{1-x^2}}{2} \right] = \sqrt{\frac{1-x}{1+x}} \qquad (3.5$$

Therefore, comparing Eq. (3.4) with Eq. (3.5) one can easily show that the locking equation is given by

$$\frac{1}{x} - \frac{\sqrt{1-x^2}}{x} \cdot \frac{\sqrt{\frac{1-x}{1+x}} + \tanh\left(nKTr\frac{\sqrt{1-x^2}}{2}\right)}{1+\frac{\sqrt{1-x}}{1+x} \tanh\left(nKTr\frac{\sqrt{1-x^2}}{2}\right)}$$

$$= \frac{1-\tan[(1-n)xKTr/2]}{1+\tan[(1-n)xKTr/2]} \qquad \ldots \quad (3.6)$$

In many practical situations the phase-difference between the local oscillator and the reference input is not very large and in this case the linearised version of the loop gives a reasonably accurate result. This is obtained when $\sin\phi$ is replaced by $\pi/2 \cdot \phi$ which can be done only when ϕ lies within $-\pi/2$ and $+\pi/2$. In such a case the governing equations of the loop are given by

$$\frac{d\phi}{dt} = \Omega - \frac{2}{\pi} K\phi, \tag{3.7}$$

and

$$\frac{d\phi}{dt} = \Omega \tag{3.8}$$

Thus the instantaneous value of the phase difference ϕ is given by $0 \leqslant t \leqslant T$

$$\phi = \phi_0 \exp\left(-\frac{2}{\pi} Kt\right) + \frac{\pi}{2} \cdot \frac{\Omega}{K}\left[1 - \exp\left(-\frac{2}{\pi} Kt\right)\right] \quad \ldots \quad (3.9)$$

and corresponding equation of lock-in can be written as

$$\left[1 - \exp\left(-\frac{2}{\pi} KT\right)\right]\left(\phi_0 - \frac{\pi}{2}\frac{\Omega}{K}\right) = \Omega\,(Tr - T) \quad \ldots \quad (3.10)$$

which on simplification yields,,

$$\frac{\Omega}{K} = \frac{1 - \exp\left(-\frac{2}{\pi} KT\right)}{1 - \exp\left(-\frac{2}{\pi} KT\right) + 4\frac{K}{w_r}(1-n)} \quad \ldots \quad (3.11)$$

where w_r is the repetition rate of interruption measured in radians per second

THE APC LOOP WITH A LOW-PASS FILTER

In this section the behaviour of the automatic phase control circuit incorporating a low-pass filter in the loop will be discussed. The operation of this circuit is different from the APC circuit without filter. This is because of the fact that during the on-period of the input signal, the instantaneous phase or frequency of the local oscillator will be gradually pulled towards that of the reference input provided the open loop frequency error is less than the maximum permissible value of the locking range. If the open loop frequency error is less than the maximum permissible value of the locking range, the frequency or the instantataneous phase of the local oscillator will be locked in synchronism with the reference input within a very short time compared to the on-period of the incoming wave. The control signal, obtained as a result of comparison of the instantaneous phases of the local oscillator and reference input is a d.c. potential that manages to maintain the synchronism between them. This d.c. potential depends on the time constant of the low-pass filter, the duty cycle of the incoming signal, open loop frequency error and the strengths of the oscillators and the reference input.

During the off-period the error signal is absent but the d.c. potential, which has been stored up in the low pass filter during the on-period, will try to hold the instantaneous phase of the local oscillator to the locked value during the off-period. However, the phase will drift away from the locked value and reach a value wihch will be governed by the duty cycle as well as by the characteristics of the filter. The local oscillator is said to be synchronised if either of the following conditions is satisfied viz.,

(i) the reduction of the phase difference between the local oscillator and the reference input during the on-period must compensate thedrift of phase difference between them during the off-period,

(ii) the phase of the local oscillator should attain a steady value at the end of the successive on-period of the input wave.

In this section an expression for the carrier locking ratio Ω/K will be derived with respect to two filters one having no limiting high frequency and the other a finite high frequency gain.

FILTER WITH NO LIMITING HIGH FREQENCY GAIN

Let us consider the APC loop with the low-pass filter as shown in Fig. 1. Assuming that the phase difference lies within $-\pi/2$ and $+\pi/2$ one can then with a resonable degree of accuracy replace $\sin\phi$ by $\frac{2}{\pi}\cdot\phi$ and write the governing equation of the linearised APC loop as

$$\tau\frac{d^2\phi}{dt^2}+\frac{d\phi}{dt}+\frac{2}{\pi}K\phi=\Omega, \tag{4.1}$$

where τ is the time constant of the filter network. The solution of which is given by

$$\phi\cong\frac{\Omega}{K}+\frac{2}{\pi}\left[\left(\phi_0-\frac{\pi}{2}\frac{\Omega}{K}\right)+\Omega\tau\right]\exp\left(-\frac{2}{\pi}Kt\right)$$

$$-\frac{2}{\pi}\left[\frac{2}{\pi}K\tau\left(\phi_1-\frac{\pi}{2}\frac{\Omega}{K}\right)+\Omega\,\tau\right]\mathrm{Exp}\left(-\frac{}{\tau}\right), \qquad \ldots \quad (4.2)$$

$$\text{for } \frac{2}{\pi}K\tau<<1$$

$$\text{and } \phi\cong\frac{\Omega}{K}+C\sin\left(\sqrt{\frac{2}{\pi}K\tau}\,\frac{t}{2\tau}+\theta\right)\exp\left(-\frac{t}{2\tau}\right),\ \frac{8}{\pi}K\tau>>1 \qquad \ldots \quad (4.3)$$

where
$$C \sin \theta = \phi_0 - \frac{\pi}{2} \frac{\Omega}{K},$$

$$C \cos \theta = \frac{\Omega + \frac{1}{2\tau}\left(\phi_0 - \frac{\pi}{2} \frac{\Omega}{K}\right)}{\sqrt{2/\pi \cdot \frac{K}{\tau}}} \qquad \dots \quad (4.4)$$

and $\phi = \phi_0$ at time $t = 0$.

In this sub-section the equation for locking ratio will be derived. This is based upon the utilisation of the linearised solutions [vide Eq. (4.2) and Eq. (4.3) of Eq. (4.1) and the first condition of section 4]. Remembering this one can easily show (see Appendix A) that the governing equation of the loop during the off-period is given by

$$\frac{d\phi}{dt} \cong \Omega - K\epsilon^{-t/\tau} \left[\frac{\Omega}{K} \cdot \frac{T}{\tau} + \frac{1}{K\tau} \left\{\left(\phi_0 - \frac{\pi}{2} \frac{\Omega}{K}\right) + \tau\Omega\right\}\right], \qquad \dots \quad (4.5)$$

$$\frac{8}{\pi} K\tau << 1$$

and

$$\frac{d\phi}{dt} \simeq \Omega - K\epsilon^{-T/t}\left[\frac{\Omega}{K}(\epsilon^{T/\tau}-1) + 4C\epsilon^{-\frac{T}{2\tau}} \sin\left(\sqrt{\frac{8}{\pi}K\tau}\,\frac{T}{2\tau} - \tan^{-1}\sqrt{\frac{8}{\pi}K\tau} - \theta\right)\right.$$

$$\left. + \frac{4C}{\sqrt{\frac{2}{\pi}K\tau}} \sin\left(\tan^{-1}\sqrt{\frac{8}{\pi}K\tau} - \theta\right)\right], \; \frac{8}{\pi} K\tau >> 1 \qquad \dots \quad (4.6)$$

The solutions of Eq. (4.5) and Eq. (4.6) are respectively given by

$$\phi(t) \simeq \Omega\,(t-T) + K\tau\left[\frac{\Omega}{K} \cdot \frac{T}{\tau} + \frac{1}{K\tau}\left\{\left(\phi_0 - \frac{\pi}{2} \cdot \frac{\Omega}{K}\right) + \Omega\tau\right\}\right]$$

$$\times \left(\exp\left(-\frac{t}{\tau}\right) - \exp\left(-\frac{T}{\tau}\right)\right) + \phi(T), \; \frac{8}{\pi} K\tau << 1 \qquad \dots \quad (4.7)$$

and

$$\phi(t) \simeq \Omega\,(t-T) + K\tau q\left(\epsilon^{-\frac{t}{\tau}} - \epsilon^{-\frac{T}{\tau}}\right) + \phi(T), \; \frac{8}{\pi} K\tau >> 1 \qquad \dots \quad (4.8)$$

where $\phi = \phi(T)$ at time $t = T$ and q stands for the right-hand side of Eq. (4.6). From these two equations one can easily show that the locking ratios Ω/K of the APC circuit with a low-pass in the loop are given by

$$\frac{\Omega}{K} \simeq \frac{\pi}{2}\left[1-\left\{\left(\epsilon^{-\frac{Tr}{\tau}}-\epsilon^{-\frac{T}{\tau}}\right)+\epsilon^{-\frac{2KT}{\pi}}\right\}\right] \Big/ \left[KTr(1-n)+\frac{\pi}{2}\, nKTr\right.$$

$$\left.\left(\epsilon^{-\frac{Tr}{\tau}}-\epsilon^{-\frac{T}{\tau}}\right)+\frac{\pi}{2}\left(\frac{\pi}{2}-KT\right)\left\{\left(\epsilon^{-\frac{T}{\pi}}-\epsilon^{-\frac{T}{\tau}}\right)+\epsilon^{-\frac{2}{\pi}KT}\right\}\right],$$

$$\text{for } \frac{8}{\pi}K\tau \ll 1 \qquad \dots \quad (4.9)$$

and for the case when $8/\pi K\tau \gg 1$

$$\frac{\Omega}{K} \simeq \frac{1-\left[\cos\left(\sqrt{\frac{2}{\pi}\cdot K\tau}\,\frac{T}{\tau}\right)+\sin\left(\sqrt{\frac{2}{\pi}K\tau}\,\frac{T}{\tau}\right)\right]\epsilon^{-T/2\tau}}{1+\left[\left(\frac{4}{\pi}K\tau-1\right)\sin\left(\sqrt{\frac{2}{\pi}K\tau}\,\frac{T}{\tau}\right)-\cos\left(\sqrt{\frac{2}{\pi}K\tau}\,\frac{T}{\tau}\right)\right]} \qquad \dots \quad (4.10)$$

FILTER WITH FINITE HIGH FREQENCY GAIN

In this sub-section carrier locking equation for the APC circuit with a low-pass filter with finite high frequency gain will be derived. This is based upon the reasonable assumption, although not a very accurate one, that at the end of the on-period of the input wave the instantaneous phase-difference ϕ attains a steady state value. The governing equations of such an APC circuit are given by (see Appendix B)

$$\frac{d\phi}{dt} \cong \Omega-\frac{2}{\pi}K\cdot\frac{1+xp\tau}{1+(1+x)p\tau}\cdot\phi \qquad \dots \quad (4.11)$$

$$\frac{d\phi}{dt} \simeq \Omega-\frac{2}{\pi}\,\frac{x}{1+x}K\phi-\frac{2}{\pi}\left[\frac{\pi}{2}\,\frac{\Omega}{K}-\frac{x}{1+x}\phi(T)\right]\epsilon^{-\frac{t-T}{(1+x)\tau}}$$

The solution of Eq. (4.11) can be written for the case when

$$\frac{8}{\pi}(1+x)K\tau > (1+\frac{2}{\pi}xK\tau)^2 \text{ as} \qquad \dots \quad (4.12)$$

$$\phi \simeq \frac{\pi}{2}\,\frac{\Omega}{K}+C\,\epsilon^{-\alpha t}\sin(\beta t+\theta) \qquad \dots \quad (4.13)$$

where C and θ are constants to be determined from the boundary conditions, namely, at time $t = 0$, $\phi = \phi_0$ and at time $t = T$, $\frac{d\phi}{dt} = 0$, α and β are given by

$$\alpha = \frac{1+\frac{2}{\pi}xK\tau}{2(1+x)\tau}$$

$$\beta = \frac{\sqrt{\frac{8}{\pi}(1+x)K\tau-(1+\frac{2}{\pi}xK\tau)^2}}{2(1+x)\tau} \qquad \dots \quad (4.14)$$

The instantaneous value of the phase difference during the off-period of the signal is given by

$$\phi \simeq \frac{\pi}{2m} \cdot \frac{\Omega}{K} + \left[\Omega - \frac{2}{\pi} mK\phi(T) \right] \left[\left[(1+x)\tau - \frac{\pi}{2mK} \cdot \epsilon^{-\frac{2}{\pi}mkT} \right] \epsilon^{-\frac{2}{\pi}mkT} \right.$$

$$\left. -(1+x)\tau \left[\Omega - \frac{2}{\pi} mK\phi(\tau) \right] \exp\left(\frac{2}{\pi} mKt - \frac{t-\tau}{(1+x)\tau} \right), \qquad \ldots \quad (4.15)$$

where m is the ratio of a.c. to d.c. gain of the filter network. Comparing (4.13) Eq. (4.14) one can write the carrier locking equation as

$$\frac{\pi}{2} \simeq \frac{\pi}{2m} \cdot \frac{\Omega}{K} + \left[\Omega - \frac{2}{\pi} mK\phi(T) \right] \left[(1+x)\tau - \frac{\pi}{2mK} \epsilon^{-\frac{2}{\pi} mkT} \right] \epsilon^{-\frac{2}{\pi} mkT}$$

$$(1+x)\,\tau \left[\Omega - \frac{2}{\pi} mK\phi(T) \right] \exp -\frac{2}{\pi} mKT\tau - \frac{T(1-n)}{(1+x)\tau} \qquad (4.16)$$

Although it is not very difficult to find an expression for the locking ratio (Ω/K) in terms of the system parameters from Eq. (4.16) it is considered reasonable to mention the conclusion of the analysis which are evident from Eq. (4.16), viz., (i) the locking ratio (Ω/K) will increase with increasing value of the repetition frequency (f), (ii) it (Ω/K) will also increase with increasing value of the duty cycle (n).

SIDE-BAND LOCKING—SIMPLE APC CIRCUIT

In this section locking phenomena in a simple APC circuit with respect to sideband components of the input signal will be discussed. Following Fraser's argument (Fraser, D. W., 1957) it is easy to show that the synchronisation of local oscillation with any one of the sideband components is feasible. It seems that locking can be achieved with a few of the sideband components around the carrier if the incoming signal to noise ratio is not high. This is because of the fact that for higher order sideband components locking range will be correspondingly small—not only because of the low value of the particular synchronising component of the input signal but also due to the effect of the $R-C$ time constant of the self-bias circuit of the oscillator (op. cit. 2). Thus for synchronisation with higher order sideband components it is logical to expect that the local oscillator due to its inherent phase jitter may sometimes step out of synchronism with respect to the particular higher order spectral component of the input signal. In view of the above fact is preferable to use a tunable high Q circuit to be followed by the APC circuit as shown in fig. 2. As the input to the high Q tuned circuit consists of a series of pulsed sinusoid so the input to the APC circuit consists of the required sideband component to which the high Q circuit has been tuned plus a few sideband components. For practical purposes one can ignore the effect of all the side-band components on the locking behaviour of the APC circuit except two adjacent

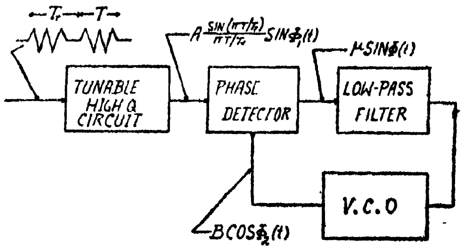

Fig. 2. Block diagram of an APC circuit preceded by a tunable high Q circuit. The high Q circuit has been assumed to be tuned to the first order sideband of the input interrupted sinusoids of on-period T_r seconds and repetition period Tr seconds.

components around the tuned component. Therefore, care should be taken to see that frequency of the local oscillator lies comparatively close to that of the wanted component of the input spectrum; otherwise, the local oscillator may unnecessarily be pulled towards the unwanted component. In ths section techniques of sideband locking of an APC circuit proceded by a high Q tuned circuit will be discussed.

Let us suppose that the carrier amplitude in relation to the duty cycle of the interrupted wave is chosen in such a way that the locking range due to the carrier component is not greater than half the repetition rate of interruption of the input wave. Let us consider the case of locking the local oscillator with the help of the first order upper sideband component of the input wave. In this case a sort of unwanted pulling of the local oscillator as stated elsewhere in the text may occur either due to the carrier component or to second order upper sideband component. Thus when the frequency of the local oscillator lies in between the carrier component and the first order upper sideband component, the governing equation of the loop can be written as

$$\frac{d\phi_{1u}}{dt} = \Omega_{1u} - xK\sin\phi_{1u} \quad \ldots \quad (5.1)$$

and

$$\frac{d\phi_{cu}}{dt} = \Omega_{eu} - y_{cu}K\sin\phi_{eu} \quad \ldots \quad (5.2)$$

where ϕ_{1u} and ϕ_{cu} are respectively the phase difference of the local oscillatoe of instanϕaneous angular frequency w_{10} with respect to the first order upper sideband comptonent and the carrier component and

$$x = \frac{\sin\left(\frac{\pi T}{Tr}\right)}{(\pi T/Tr)}, y_{cu} = \frac{(T/Tr)}{\sqrt{1+4Q^2\left(\frac{\omega_r}{\omega_c-\omega_r}\right)^2}} \quad \ldots \quad (5.3)$$

Again the corresponding loop equation for the case when the frequency of the local ostICllator lies between the first order upper sideband component and the second order upper sideband component of the input wave can be written as

$$\frac{d\phi_{1u}}{dt} = \Omega_{1u} - xK \sin\phi_{1u}, \qquad \ldots \quad (5.4)$$

and

$$\frac{d\phi_{2u}}{dt} = \Omega_{2u} - y_{2u} K \sin\phi_{2u}, \qquad \ldots \quad (5.5)$$

where ϕ_{2u} is the phase difference of the local oscillator with r3mspect to the second order upper sideband component of the input wave and

$$y_{2u} = \frac{\sin\left(\frac{2\pi T}{T_r}\right) \Big/ \left(\frac{2\pi T}{T_r}\right)}{\sqrt{1+4Q^2\left(\frac{\omega_r}{\omega_c - \omega_r}\right)^2}} \qquad \ldots \quad (5.6)$$

Now for the case when the frequency of the local oscillator lies between the carrier and the first order upper sideband, in order to avoid pulling by the carrier component, it is desirable to satisfy the following conditioons

$$\Omega_{lu} < xK,$$

$$\Omega_{cu} > y_{cu}. \qquad \ldots \quad (5.7)$$

Similarly for the case when the frequency of the local oscillator lies between the first order and second order upper upper sideband components of the input wave it is advisable to choose the loop parameters in such a way as to satisfy the following conditions

$$\Omega_{1u} < xK,$$

$$\Omega_{2u} > y_{2u}K \qquad \ldots \quad (5.8)$$

Therefore, the maximum possible locking range for the first order sideband component is given by

$$\Omega_{1u} \simeq K \frac{\sin\left(\frac{\pi T}{T_r}\right)}{(\pi T/T_r)} \qquad \ldots \quad (5.9)$$

APC CIRCUIT INCORPORATING A LOW-PASS FILTER AND PRECEDED BY A HIGH Q TUNED CIRCUIT

In this section the effect of incorporating a low-pass filter in the loop on the sideband locking behaviour of an APC circuit when it is proceded by a tunable high Q circuit will be discussed. Inclusion of the low-pass filter in the loop will, however, introduces the so-called pull-in phenomena (Chakrabarti, *et al.* 1964). The response to transients may no longer be deadbeat even if the initial difference frequency lies within the locking range and instaneaneous frequency may drift a few beat cycles of continuous decreasing frequency till equillibrium is reached. It has been shown (op. cit.) that in such a case the locking range of the APC circuit depends on the average gain of the filter network over the range d.c. to open loop frequency difference. Analytical expressions of locking range for different components can be written as

$$\Omega_{1u} \simeq K \frac{\sin\left(\frac{\pi T}{T_r}\right)}{\frac{\pi T}{T_r}} \bar{G}(\Omega_{1u}), \qquad \dots (6.1)$$

$$\Omega_{2u} \simeq y_{2u} \bar{G}(\Omega_{2n}), \qquad \dots (6.2)$$

$$\Omega_{cu} \simeq y_{cu} \; G(\Omega_{cu}) \qquad \dots (6.3)$$

Therefore following the arguments of section 5, one can show that either of the following conditions is to be satisfied in order to avoid unnecessary pulling by the unwanted component

$$x \bar{G}(\Omega_{1u}) > y_{2u}\bar{G}(\Omega_{2u}) \qquad \dots (6.4)$$

$$x\bar{G}(\Omega_{1u}) > y_{cu}\bar{G}(\Omega_{cu}) \qquad \dots (6.5)$$

EXPERIMENTAL SET UP AND RESULTS

In this section we shall describe experimental set-up and discuss the experimental results in relation to the conclusions of the analysis presented in the text.

Fig. 3 shows the experimental set-up for taking measurements on the variation of locking range with the duty cycle of the input interrupted wave and for the

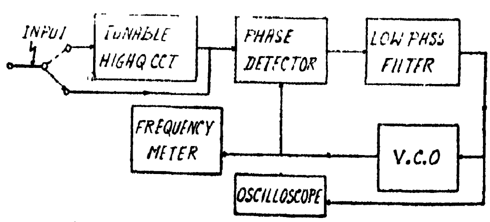

Fig. 3. Experimental set-up for taking measurements on the performance of the APC circuit with respect to an input interrupted sinusoids. For sideband locking the switch is to be placed at the dotted position.

side-band locking high Q-tuned circuit is to be used before the conventinal APC loop. The circuit diagram of the high Q-tuned circuit is shown in the

Fig. 4. The interrupted wave is obtained by means of an electronic switch. As discussed in the text in order to observe accurately the phenomenon of

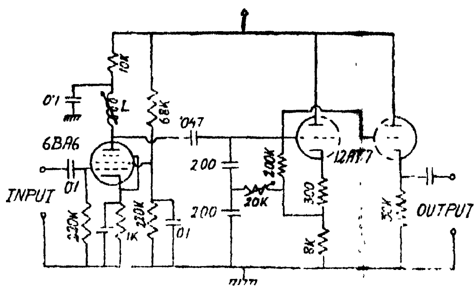

Fig. 4. The circuit diagram of a tunable high Q circuit. Tuning can be adjusted by the variable inductance L and the Q value of this tuned circuit can be continuously adjusted by the 20K potentiometer. In practice this 20K potentiometer consists of two potentiometers —one for coarse control and the other for fine adjustment.

sideband locking of the voltage controlled oscillator it is necessary that the locking range of the VCO for sideband component with respect to half the separation between the sidebands should be small. This, in effect, requires that the frequency stability of the local oscillator should be good. For this reason a Clapp's oscillator with a varactor (V 33) for its control of frequency has been used for the observation of sideband locking. Otherwise, there is possibility that the system may fall out of synchronism with respect to the particular sideband component of the input signal. This is because of the fact that the locking range of the particular sideband component is required to be small in order to avoid unnecessary pulling by the neighbouring sideband component. Thus the frequency instability of the local oscillator must be small compared to the locking range of the system for the particular sideband component of interest. The local oscillator with the varactor (V-33), which is known to have a reasonable frequency stability, is shown in Fig. 5. Care should be taken to see that the input amplifier feeding the phase detector should have a flat top response. Presence

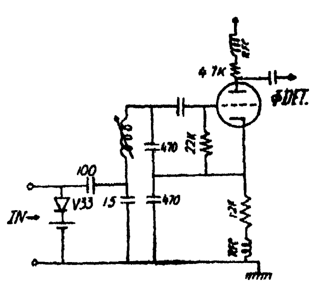

Fig. 5. A typical circuit diagram of a Clapp's oscillator with the varactor diode V-33 connected (with proper bias voltage) across the 15 pF capacitor for controlling the instantaneous frequency of the oscillator by means of a d.c. voltage to be applied in between the input terminals.

of dip anywhere in the characteristics may give rise to suprious effects (Biswas, 1964).

Fig. 6. shows the variation of the locking range with the ratio of the on-period to the off-period for two different carrier levels. From this curve one can conclude

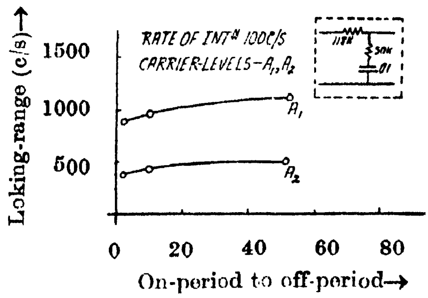

Fig. 6. Graphical representation of the variation of the locking range of the APC circuit with ON-OFF ratio of the input interrupted wave having carrier levels A_1 and A_2 and a repetition rate of 100 c/s. A_1 is greater than A_2. The filter network of the APC loop is inset in the figure.

that the locking range of the phase locked oscillator with an interrupted wave input approaches to that of the phase locked oscillator with a CW input. Fig. 7

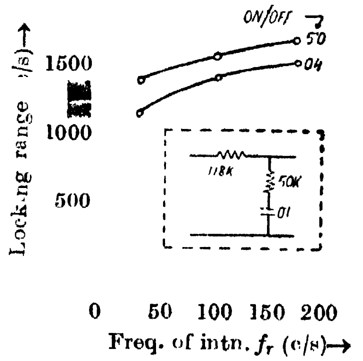

Fig. 7. Graphical representation of the dependence of the Locking Range of the APC circuit on the variation of the Frequency of interruption of the input wave for two different values of the on-off ratios of the input wave. The filter network is inset in the diagram.

gives a graphical representation of the variation of the locking range with repetition rate of the input wave for two different values of the duty cycle of the input. It is to be noted that these results are in good agreement with the conclusions of the analysis of section 4.2. The experimental results regarding the sideband locking of the phase locked oscillator is shown in the following Table I.

TABLE I

Centre frequency of the reference input 410 Kc/s; on/off ratio of the input wave is unity

Detuning of the reference input	Observed repetition frequency for locking	Inference from the observation for sideband locking experiment for different values of the carrier levels
+ 1000 cycles/second + 750 cycles/second	Nearly 1000 cycles/sec. Nearly 750 cycles/sec.	The sideband locking experiment has been repeated for different carrier levels and it has been found that lower carrier levels give better results on the observation of sideband locking.

CONCLUSIONS

The phenomenon of locking of an APC circuit with an interrupted wavehave been discussed fully. Locking range equation (vide Eq. 3.11, Eq. 4.10 and Eq. 4.16) have been derived. These equations are simple as such simple analytical or graphical solutions are possible. Experimental results presented in this paper give an useful information regarding the variation of locking range of the VCO with the duty cycle of the input wave and the phenomenon of sideband locking technique of the APC circuit when it is preceded by a high Q tuned circuit. Experimental results are in good agreement with the simplified analysis presented in text. It has been found that the synchronisation of the local oscillator with any of the sideband components of the input wave becomes easier when it is preceded by a high Q tuned circuit.

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APPENDIX

A. *Derivation of the Loop Equation of the APC circuit with a Low Pass Filter with No Limiting High Frequency gain during the off-period of the Input Wave*

On comparison of Eq. (2.3) with Eq.)(2.4) one can write for the output of the low pass filter with no finite high frequency gain at the end of the on-period of the input wave as

$$e_d(t) = \frac{\epsilon^{-t/\tau}}{\tau} \int_0^T \epsilon^{t_1/\tau} e(t_1)\, dt_1,$$

$$\simeq \frac{2}{\pi} \frac{\mu}{\tau} \epsilon^{-t/\tau} \int_0^T \phi(t_1) \epsilon^{t_1/\tau}\, dt_1. \qquad \ldots \quad \text{(A.1)}$$

Comparing Eq. (A.1) with Eq. (4.2) and Eq. (4.3) one can show that can be written as

$$e_d(t) \simeq \frac{2}{\pi} \mu \epsilon^{-t/\tau} \left[\frac{\pi}{2} \frac{\Omega}{K} \frac{T}{\tau} + \frac{\pi}{2K\tau} \left\{ \left(\phi_0 - \pi/2 \frac{\Omega}{K} \right) + \Omega\tau \right\} \right]$$

for

$$\frac{8}{\pi} K\tau << 1 \qquad \ldots \quad \text{(A.2)}$$

and

$$e_d(t) \simeq \mu \epsilon^{-t/\tau} \left[\frac{\Omega}{K} \left(\epsilon^{\frac{T}{\tau}} - 1 \right) + \frac{4C\epsilon^{-\frac{T}{2\tau}}}{\pi\sqrt{\frac{2}{\pi} K\tau}} \sin\left(\sqrt{\frac{8}{\pi} K\tau}\, \frac{T}{2\tau} + \theta - \tan^{-1}\sqrt{\frac{8}{\pi} K\tau\pi} \right) \right.$$

$$\left. + \frac{4C}{\pi\sqrt{\frac{2}{\pi} K\tau}} \sin\left(\tan^{-1} \sqrt{\frac{8}{\pi} K\tau} - \theta \right) \right], \text{ for } \frac{8}{\pi} K\tau >> 1 \qquad \ldots \quad \text{(A.3)}$$

Substitution of $e\,(t)$ from (A.2) and A.3) in Eq. (2.2) yields the loop equation for the off-period of the input signal for the APC circuit with the low-pass filter.

B. *Derivation of the Governing Equation of the Loop during the off-period of the Input wave for the Filter with finite high Frequency Gain*

Let us consider the low pass filter with limiting high frequency gain as shown in Fig. In this way it can be shown that the impulse response of the network is given by

$$f(t) = \frac{x}{1+x} \delta(t) + \frac{1}{\tau(1+x)^2} \exp\left(-\frac{t}{(1+x)\tau} \right), \qquad \ldots \quad \text{(B.1)}$$

where $\delta(t)$ is the Dirac's delta function. Therefore the output of the filter at the end of the on-period of the filter network is given by

$$e_d(t) \cong \mu \frac{2}{\pi} \frac{x}{1+x} \phi(t) + \frac{2}{\pi} K_1 \exp\left(- \frac{t}{(1+x)\tau} \right), \qquad \text{... (B.2)}$$

where K_1 is a constant to be found frrom the boundary condition. Therefore the governing equation of the loop during the off-period is given by

$$\frac{d\phi}{dt} = \Omega - \frac{2}{\pi} K_1 \frac{x}{1+x} \phi(t) - \frac{2}{\pi} KK_2 \exp\left(- \frac{t}{(1+x)\tau} \right), \qquad \text{... (B.3)}$$

where K_2 is a constant to be determined from the boundary condition at time $t = T$, $\frac{d\phi}{dt} = 0$.